Testing of Hypothesis

(Paired Sample t-test)

Dr. D S Dhakre

Visva Bharati, Sriniketan, West Bengal, India

Example:

Ten employees of a firm were given a typing-speed test before and after a one-month training program. Do the scores give evidence that the employees benefited from the training?

Employee	A	B	C	D	E	F	G	H	I	J
Test 1 (X_i)	8	6	5	9	10	7	8	6	7	9
Test 2 (Y_i)	9	8	7	10	12	7	9	8	8	12

Solution:

Testing the significance of the difference between the means of two dependent samples where $n_1 = n_2 = n$, and the samples are not independent but related, we follow the paired t-test procedure.

Let μ_1 be the mean of X_i, μ_2 the mean of Y_i, $d_i = X_i - Y_i$, and μ_d the mean of the differences d_i in the population.

We want to test

$H_0: \mu_1 = \mu_2$ or $\mu_d = 0$

against $H_a: \mu_1 < \mu_2$ or $\mu_d < 0$

at α level of significance.

The test statistic is

$$t = \frac{|\bar{d} - \mu_d|}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$$

where $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$ and

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(d_i - \bar{d})^2 =$$

$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} d_i^2 - \frac{(d_i)^2}{n}\right)$$

Table for calculating sample mean square

Test 1	Test 2	$d_i = (X_i - Y_i)$	d_i^2
8	9	−1	1
6	8	−2	4
5	7	−2	4
9	10	−1	1
10	12	−2	4
7	7	0	0
8	9	−1	1
6	8	−2	4
7	8	−1	1
9	12	−3	9
Total		**−15**	**29**

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$$

$\bar{d}$ = −15 / 10 = −1.5

$$s^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} d_i^2 - \frac{(d_i)^2}{n}\right)$$

$$s^2 = \frac{1}{10-1}\left(29 - \frac{(-15)^2}{10}\right)$$

s^2= 0.7222

$s = \sqrt{0.7222} = 0.850$

Under H_0, the calculated value of the test statistic

$$t = \frac{|\bar{d} - \mu_d|}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$$

$$t = \frac{|-15|}{\frac{0.850}{\sqrt{10}}}$$

t = **−5.582** ($|t_{cal}|$ = 5.582)

Decision rule:

Based on the t-table for $df = n - 1 = 9$ and $\alpha = 0.05$, we have $t_{9,\,0.05}$ = 1.83 (t_{tab}). Since $|t_{cal}|$ (5.582) > t_{tab} (1.83), H_0 is rejected at 5% level of significance. This indicates that the improvement in typing speed after the training program is statistically significant, suggesting that the employees benefited from the training.

Do your self

Example 1: Rice Yield Before and After Biofertilizer

Farm	Before	After
1	4.2	4.7
2	4.5	4.9
3	4.4	4.8
4	4.6	5.0
5	4.3	4.7
6	4.5	4.9
7	4.4	4.8
8	4.6	5.1

Question

Did the biofertilizer significantly increase yield?

Example 2: Plant Height Before and After Growth Regulator

Plant	Before (cm)	After (cm)
1	45	50
2	48	53
3	46	52
4	47	54
5	49	55
6	44	49
7	46	51
8	48	54

Question

Did the growth regulator significantly increase plant height?